Adjoint Technique for Sensitivity Analysis of Coupling Factors According to Geometric Variations

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A physically equivalent electric network and its parameters are extracted from a 3D FE model formulated by the Darwin's approximation to Maxwell's equations. In this paper we add a sensitivity analysis approach to the extraction of inductive parasitics. The adjoint technique is applied to the sensitivity analysis of extracted partial inductances and coupling factors. The resulting algorithm is an efficient way to calculate gradient information with respect to a large number of parameters. In an example, we use the gradients to guide the geometrical optimization of an EMC filter.

Index Terms—**Adjoint technique, Mutual coupling, Sensitivity analysis, Finite element modelling, Model reduction**

I. Introduction

At first, a purely functional electromagnetic compatibility (EMC) filter design does not account for parasitics. Later on, the missing parasitics are modelled by additional inductances and capacitances for which the parameters can be calculated from a finite-element (FE) model. Some of the additional circuit elements are then identified as being responsible for the sub-optimal filter performance [1]. Finally, the question arises, which geometric parameters need to be adapted in order to improve the filter performance.

In [2] an automated extraction method for parasitic elements has been proposed, which retains a physical interpretability of the complete network. The ability to interpret the network distinguishes this approach from the more common model order reduction (MOR) [3]. Although the partial element equivalent circuit (PEEC) technique [4] [5] leads to interpretable circuit models, the introduced approach requires much less physically relevant parameters, independent of the chosen discretization. Adjoint sensitivity analysis was already demonstrated with the parametric model order reduction [6] and for partial inductances in PEEC [7].

Based on the former ansatz, we present the extraction and sensitivity analysis of partial inductances and coupling factors.

II. Extracting Partial Inductances and Coupling Factors

In order to compute coupling factors, the self- and mutual-inductances of the 3D geometry are extracted by comparing field simulation results with electric circuit parameters. The most suitable approach to extract equivalent electric circuits is the Darwin model, an approximation to the Maxwell's equations, that does not include wave propagation (see also [8] and [9]). Using the finite element (FE) method, the system of differential equations, describing the Darwin model leads to the linear system of equations:

$$\left(\mathbf{A} + s^2\mathbf{B}\right)\mathbf{x} = \mathbf{y}\,. \tag{1}$$

Where,

$$\mathbf{A} = \begin{pmatrix} \varepsilon\triangle & 0 \\ 0 & 0 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} -\varepsilon^2\mu & \varepsilon\nabla\cdot \\ \varepsilon\nabla & -\nabla\times\mu^{-1}\nabla\times \end{pmatrix} \tag{2}$$
$$\mathbf{x} = \begin{pmatrix} s^2\varphi \\ \mathbf{E}_\sigma \end{pmatrix} \qquad \mathbf{y} = s^3\begin{pmatrix} \varepsilon g \\ -\nabla\mu^{-1}g \end{pmatrix}$$

Here, ε is the electric permittivity, μ the magnetic permeability, $\mathbf{E}_\sigma$ is the electric field that is related to internal currents, and g is an auxiliary field, which is computed by solving the following equation:

$$-\,\mu^{-1}\Delta g = \nabla\cdot\mathbf{j_s}\,, \tag{3}$$

with $\mathbf{j}_s$ being the injected source current density, which is related to the network current matrix $\mathbf{I}$.

From (1) the field solution $\mathbf{x}$ is computed and the impedance matrix $\mathbf{Z}$ of the electric network is determined:

$$\mathbf{Z}(s) = s^{-2}\left(\mathbf{P}\mathbf{x}\right)\mathbf{I}^{-1}\,. \tag{4}$$

The projection operator $\mathbf{P}$ links the FE potentials φ to potentials at vertices of the network model. The impedance matrix $\mathbf{Z}$ is related to the inductance $\mathbf{L}$ and the capacitance $\mathbf{C}$ by

$$\mathbf{Z}(s) = \left((s\mathbf{L})^{-1} + s\mathbf{C}\right)^{-1}\,. \tag{5}$$

By computing impedance matrices at multiple frequencies well below the first resonance of the system, a least squares fit allows to extract the inductance.

The coupling factor between two partial inductances (i,j) can then be calculated with the self-inductances L_i and L_j and the mutual inductance L_{ij} by:

$$K_{ij} = L_{ij}\left(|L_i|\cdot|L_j|\right)^{-\frac{1}{2}}\,. \tag{6}$$

III. Adjoint Sensitivity Method

In order to calculate the sensitivities of the inductances, the change of the impedance $\mathbf{Z}$ with respect to a model parameter p_i is calculated by

$$\frac{\partial\mathbf{Z}}{\partial p_i} = \left(\frac{\partial\mathbf{Z}}{\partial\mathbf{x}}\right)^T\frac{\partial\mathbf{x}}{\partial p_i} \tag{7}$$

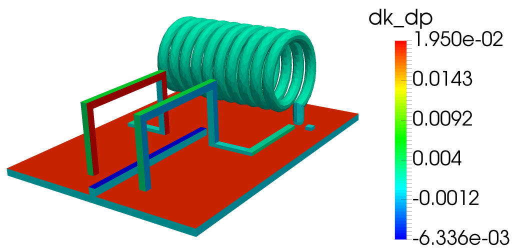

Fig. 1. Sensitivity map of the inductive coupling factor K between the two capacitors (initial geometric design) with regards to the displacement along face normals p.

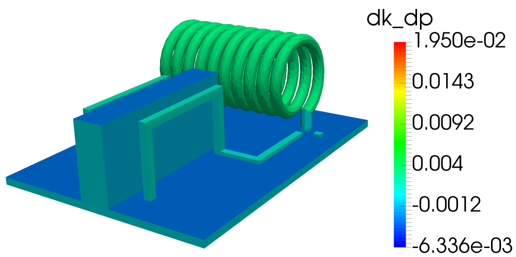

Fig. 2. Sensitivity map of the inductive coupling factor K between the two capacitors (improved design) with regards to the displacement along face normals p.

The second term represents the sensitivity of the solution vector $\mathbf{x}$ on the model parameter p_i and follows from Eq. (1), i.e.,

$$\frac{\partial \mathbf{x}}{\partial p_i} = \left(\mathbf{A} + s^2\mathbf{B}\right)^{-1} \left[\frac{\partial \mathbf{y}}{\partial p_i} - \left(\frac{\partial \mathbf{A}}{\partial p_i} + s^2 \frac{\partial \mathbf{B}}{\partial p_i}\right)\mathbf{x}\right]. \quad (8)$$

The adjoint technique requires the so-called adjoint solution λ, where λ needs to be computed only once for each circuit parameter:

$$\left(\mathbf{A} + s^2\mathbf{B}\right)^T \lambda = \frac{\partial \mathbf{Z}}{\partial \mathbf{x}}. \quad (9)$$

From that, one finds:

$$\frac{d\mathbf{Z}}{dp_i} = \lambda^T \left[\frac{\partial \mathbf{y}}{\partial p_i} - \left(\frac{\partial \mathbf{A}}{\partial p_i} + s^2 \frac{\partial \mathbf{B}}{\partial p_i}\right)\mathbf{x}\right]. \quad (10)$$

The adjoint technique avoids costly matrix inversions. The matrices $\partial\mathbf{A}/\partial p_i$ and $\partial\mathbf{B}/\partial p_i$ have to be computed for every parameter, but are very sparse. This method provides an efficient way to compute the sensitivity of a few quantities with respect to a much larger number of 3D model parameters.

Application Example

In Fig. 1, a first geometric design of a π-filter is shown, consisting of an inductor and two capacitors, which are modeled by equivalent loops in the 3D model. Their capacitance is added later in the network model. Its transmission coefficient is sub-optimal (blue curve versus green curve in Fig. 3) and this can be attributed to a parasitic inductive coupling between the capacitors in the geometry. It appears as a parasitic inductance in the circuit, extracted from the field model of the initial geometric design. Neglecting this coupling would improve the transmission by about 20dB (red curve). This motivates adding a shield between both capacitors (Fig. 2). The sensitivity of the transmission coefficient with respect to the shield's height and width is used as gradient information for the optimization procedure. An optimized design which reduces the parasitic coupling has been found after 19 iterations (black curve in Fig. 3).

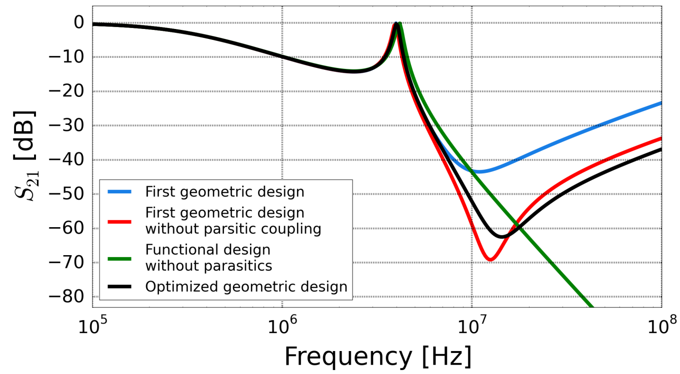

Fig. 3. Transmission of the π-filter, transmission of Fig. 1 (blue), transmission of Fig. 2 (black), transmission of ideal filter (green), transmission with coupling turned off (red)

IV. Conclusions

In this paper we presented a numerically efficient sensitivity analysis of partial inductances and coupling factors to large numbers of geometry or material parameters. This is part of an iterative optimization scheme, which retains interpretability at all times. In an example application, we show how to use the gradients from the adjoint sensitivity analysis to modify the geometry of an EMC filter in order to optimize its behavior.

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